On the Optimal Placement of Wavelength Converters in Wavelength-Routed Networks*

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Abstract

In this paper, we consider the problem of optimally placing a given number of wavelength converters on a path to minimize the call blocking probability. Using a simple performance model, we first prove that uniform spacing of converters is optimal for the end-to-end performance when link loads are uniform and statistically independent. We then show that significant gains are achievable with optimal placement compared to random placement. For non-uniform link loads, we provide a dynamic programming algorithm for the optimal placement and compare the performance with random and uniform placement. Optimal solutions for bus and ring topologies are also presented.

1 Introduction

The potential benefits of wavelength conversion in wavelength-routing networks have been studied through different traffic models [2, 3, 4, 5] recently. All-optical wavelength conversion is expensive and this has led to some recent focus on networks with sparse or limited wavelength conversion [5, 6]. In this paper, we consider the following problem. Given a network topology, a certain number of converters, and traffic statistics between the nodes, the problem of interest is the optimal placement of converters in the network. To our knowledge, this problem has not been addressed earlier. There are several approaches to this problem depending on the optimality criterion and the traffic model.

In this paper, we assume a dynamic traffic model in which connections arrive and depart from the network in a random manner and the goal is to place the given number of converters in the network such that the network call blocking probability is minimized. There are several factors which affect the optimal solution to the converter placement problem. Intuition would suggest that converters be placed at nodes which process the highest amount of transit traffic. However, placing a converter at a node that has a high transit traffic rate but does very little mixing (or switching) of traffic may not be the optimal strategy. On the other hand, if the transit traffic rate at a node is very low, then the optimal strategy may not place a converter at that node even if it mixes a significant amount of traffic. Furthermore, the distances between converters are likely to affect the optimal placement. As the distance between converters increases, the blocking probability increases. Since the number of available converters is limited, a judicious placement of converters is necessary.

The rest of this paper is organized as follows. In Section 2, we consider the placement of wavelength converters on a path assuming link load independence. In Section 3, we consider a bus network. The low connectivity of the bus network induces a strong correlation among link loads. This is taken into account by modifying the performance model accordingly. The solution in Section 3 can also be used to optimally place converters in a path of an arbitrary network where link load independence is not justified. In Section 4, we extend the solution for the path to obtain the optimal converter placement for the ring topology. Conclusions are presented in Section 5.

2 Converter Placement in a Path

In this section, we consider a path of length $H$ with negligible correlation between link loads. We start by considering an end-to-end call on a path of length $H$.

The number of wavelengths on each link is assumed to be $F$ and each link has a single fiber. Let the nodes along the path be numbered $0, 1, \ldots, H$, let the link from node $i$ to node $i+1$ be labeled as link $i$, and let the link loads per wavelength be $\rho_i$, $i = 0, \ldots, H - 1$. That is, the probability that a given wavelength is occupied on link $i$ is $\rho_i$, and the wavelength occupancy events are assumed to be statistically independent of other wavelengths on the same link and on other links. This is the binomial model of [2].

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2.1 Uniform Link Loads

First suppose that all the $\rho_i$’s are equal to $\rho$. The goal is to place $K$ converters among the $H - 1$ intermediate nodes of the path so that the blocking probability of an end-to-end call (a call from node 0 to node $H$) is minimized.

Define a segment to be the set of links between two consecutive converter nodes. Let $l_i$ be the hop-length of the $i$th segment, $i = 0, \ldots, K$ (nodes 0 and $H$ can be assumed to contain dummy converters). Let $L_K = (l_0, l_1, \ldots, l_{K-1}, l_K)$, where $\sum_{i=0}^{K} l_i = H$, be the vector denoting the hop-lengths of the $K+1$ segments, called the length vector henceforth. The success probability of the call in a sub-segment of length $l_i$, denoted by $f(l_i)$, is given by

$$f(l_i) = 1 - (1 - \bar{\rho})^{l_i}$$

(1)

where $\bar{\rho} = 1 - \rho$.

The success probability of the end-to-end call when the length vector is $L_K$ is denoted by $S(L_K)$, and is given by $S(L_K) = \prod_{i=0}^{K} f(l_i)$.

Let the length vector in the optimal placement be $L_K^{\text{opt}}$, i.e., $S^{\text{opt}}(H, K) = S(L_K^{\text{opt}}) = \max S(L_K)$, where $S^{\text{opt}}(H, K)$ denotes the success probability of an end-to-end call when $K$ converters are optimally placed along an $H$-hop path. Suppose that $(K + 1)$ divides $H$. We show below that $l_i^{\text{opt}} = H/(K + 1)$, $i = 0, 1, \ldots, K$, is the optimal length vector. To prove this, we need to show that $\prod_{i=0}^{K} f(l_i^{\text{opt}}) = [f(H/(K+1))]^{K+1} \geq S(L_K)$ for any feasible $L_K$. Equivalently, we must show that $\ln f(H/(K+1)) \geq \frac{1}{K+1} \sum_{i=0}^{K} \ln f(l_i)$ where the $l_i$’s are the elements of a feasible length vector. This follows from the fact (which can be easily proved) that $\ln f(l)$ is a concave function of the continuous variable $l \in (0, H)$. We remark here that the concavity of $\ln f(l)$ is sufficient but not necessary for the optimality of uniform placement.

The success probability of an end-to-end call under optimal placement is therefore

$$S^{\text{opt}}(H, K) = \left[f \left( \frac{H}{K+1} \right) \right]^{K+1}$$

(2)

It must be noted that this expression is exact only if $(K + 1) | H$. When this is not the case, the integral constraint on the segment lengths will make the expression in (2) an upper bound on the actual success probability. In that case, the optimal strategy is to place the converters as uniformly as possible, i.e., converters are placed so that there are $y = H - a(K + 1)$ segments of length $a + 1$ and $K + 1 - y$ segments of length $a$ where $a = \left\lfloor \frac{H}{K+1} \right\rfloor$.

2.1.1 Random Placement

We now derive a recurrence relation for the success probability of the end-to-end call when the $K$ convert-

ers are placed randomly on the path, i.e., each of the $\binom{H-1}{K}$ converter placement configurations is equally likely. Let us define $S_r(H, K)$ to be the average success probability of an end-to-end call when $K$ converters are randomly placed along a path of length $H$. Then, by conditioning on the position of the converter that is placed at the lowest node index, we obtain the recursive relationship

$$S_r(H, K) = \sum_{i=1}^{H-K} f(i) S_r(H-i, K-1) \binom{H-i-1}{K-1} \binom{K-1}{H-1}$$

(3)

for $1 \leq K \leq H - 1$. Obviously $S_r(H, 0) = f(H)$.

To see the benefits of optimal converter placement with respect to random placement, we plot the end-to-end blocking probability ($P_b$) as a function of $\rho$ for $H = 10$ and $F = 10$ and $K = 1, 2$ in Figure 1. (Unless otherwise specified, all numerical results presented in this paper are for $H = 10$ and $F = 10$.) We observe that the performance with 2 converters randomly placed is poorer than the performance with 1 converter optimally placed. Also notice that the performance difference between optimal and random is slightly enhanced as $K$ increases from 1 to 2.

![Figure 1: $P_b$ vs. $\rho$ for optimal/uniform (O) and random (R) placement with $K = 1$ and 2 converters.](image)

The performance effect of the number of converters can be seen more clearly in Figure 2 where $P_b$ is plotted against $K$. Notice that conversion plays a significant role in improving the blocking probability. This is of course due to link load independence and the long path length, as predicted by previous models [2, 4, 6]. More importantly, it can be seen that there is a large improvement in performance between random and optimal placement, reaching a maximum
of about 2 orders of magnitude at \( K = 4 \). As expected, both curves converge at \( K = 0 \) and \( K = H - 1 \). Notice that the curves also converge at \( K = H - 2 \). This is because random placement of \( H - 2 \) converters produces \( K \) segments of length 1 and one segment of length 2. Since link loads are uniform, the location of this 2-hop segment does not matter and the performance is exactly the same as obtained by optimally placing \( H - 2 \) converters. The effect of the length of the maximum segment on \( P_b \) is worth mentioning here. Consider the \( O \) curve. The length of the maximum segment under optimal (uniform) placement as \( K \) increases from 0 to 9 are, respectively, 10, 5, 4, 3, 2, 2, 2, 2, 2, 2, 1. It can be seen from Figure 2 that there is a significant performance change when the maximum segment length changes, and the performance improvement is only marginal when the maximum segment length does not change (from \( K = 4 \) to 8).

![Figure 2: \( P_b \) vs. \( K \) for optimal/uniform (O), and random (R) placement of \( K \) converters, for uniform link loads (\( \rho = 0.1 \)).](image)

Another metric to study the importance of converter placement is the utilization gain, \( G_u^* \) [2]. Let the target blocking probability \( P_b^* \), and the number of converters \( K \) be given. Let \( \rho_0 \) be the maximum load per link per wavelength achievable for the given \( P_b^* \) with the converters optimally placed, and let \( \rho_r \) be the maximum load achievable with the converters randomly placed. Then the utilization gain is defined as \( G_u^* \overset{\text{def}}{=} \frac{\rho_r}{\rho_0} \). \( G_u^* \) is plotted as a function of \( K \) in Figure 3. The gain can be as high as 1.6 for \( K = 4 \) and \( P_b = 10^{-3} \). This curve was obtained by numerically solving for \( \rho \) in (3) to obtain a \( P_b \) that is within 1% of the target \( P_b \).

![Figure 3: \( G_u^* \) vs. \( K \) for \( P_b \approx 10^{-3} \).](image)

### 2.2 Non-uniform Link Loads

We next consider the case when the link loads \( \{\rho_i\} \) are not all equal. Clearly, the optimal placement is, in general, non-uniform in this case. Before we provide an optimal solution, we make a straightforward modification of (3) to obtain an expression for random placement.

#### 2.2.1 Random Placement

We change some of the earlier definitions as follows. Define the subpath from node \( i \) to node \( j \) comprising links \( i, i + 1, \ldots, j - 1 \) to be the chain \([i, j]\). Let \( S_r(i, j, k) \) denote the success probability of an end-to-end call in the chain \([i, j]\) with \( k \) converters randomly distributed among the \( j - 1 - i \) intermediate nodes. Then, a recurrence relation for \( S_r(i, j, k) \) can be obtained as

\[
S_r(i, j, k) = \sum_{l=i+1}^{j-k} S_r(i, l, 0) S_r(l, j, k-1) \frac{(j-l-1)}{(j-i-1)} \tag{4}
\]

for \( k = 1, 2, \ldots, j - 1 - i \) and

\[
S_r(i, j, 0) = f(i, j) \overset{\text{def}}{=} 1 - \left( 1 - \prod_{m=i}^{j-1} \rho_m \right)^F \tag{5}
\]

is the success probability in the subsegment\(^2\) from node \( i \) to node \( j \). Note that this subsegment contains the links \( i, i + 1, \ldots, j - 1 \). The success probability of an end-to-end call when \( K \) converters are randomly placed on the path is simply given by \( S_r(0, H, K) \).

\(^2\)According to our terminology here, a chain may contain converters while a segment does not.
2.2.2 Optimal Placement

When the link loads are not equal, there does not appear to be a closed-form expression for the segment lengths as in the uniform link load case. We obtain a solution based on dynamic programming along the lines of [7] for this case. In [7], the problem of placing erasure nodes in DQDB networks so that slot reuse is maximized is considered and optimal solutions based on dynamic programming are presented. The nature of the objective function in our case is different from theirs; however, their approach can be modified to solve the problem at hand, as we will see below.

For any integer \( m \), define converter placement vector \( \mathbf{a} = (a_1, a_2, \ldots, a_m) \) with \( 0 < a_i < a_{i+1} \leq H \), \( i = 1, \ldots, m - 1 \). The entries of \( \mathbf{a} \) denote the placement of converters among the nodes \( 1, 2, \ldots, H \), i.e., \( a_i \) is the location of the \( i \)th converter. The reason for the inclusion of node \( H \) as a possible converter location will be apparent below. Also define \( \beta(j, \mathbf{a}) \) to be the probability that an end-to-end call is successful in chain \([0, j]\) when \( \mathbf{a} \) is the placement vector, and define the set of all converter placement vectors with the \( m \)th converter at node \( j \) as

\[
\Theta(m, j) \overset{\text{def}}{=} \{ \mathbf{a} \in Z_+^m : 0 < a_i < a_{i+1} < a_m = j, \ i = 1, \ldots, m - 2 \}.
\]

Then \( \gamma(m, j) \overset{\text{def}}{=} \max_{\mathbf{a} \in \Theta(m, j)} \beta(j, \mathbf{a}) \) denotes the success probability in the chain \([0, j]\) with the first \( m - 1 \) converters optimally placed among nodes \( 1, \ldots, j - 1 \), and the \( m \)th converter at node \( j \). Let us also define

\[
\Gamma(m, j, k) \overset{\text{def}}{=} \{ \mathbf{a} \in \Theta(m, j) : a_{m-1} = k \}.
\]

We can obtain \( \gamma(m, j) \) using the following dynamic programming procedure.

\[
\gamma(m, j) = \max_{m-1 \leq i < j} \max_{\mathbf{a} \in \Gamma(m, j, i)} \beta(j, \mathbf{a}) = \max_{m-1 \leq i < j} \gamma(m-1, i) f(i, j)
\]

for \( 2 \leq m \leq K + 1, \ m \leq j \leq H \). Clearly \( \gamma(1, j) = f(0, j) \) for \( 1 \leq j \leq H \). \( f(i, j) \) is as defined in 5.\)

The above recurrence relation directly leads to an \( O(H^2 K) \) dynamic programming algorithm for obtaining the optimal placement vector. The optimal placement vector is the solution obtained when \( K + 1 \) converters are placed among nodes \( 1, 2, \ldots, H \) such that the \( (K + 1) \)th converter is at node \( H \), i.e., \( \mathbf{a}^{\text{opt}} = \arg \max_{\mathbf{a} \in \Theta(K+1, H)} \beta(H, \mathbf{a}) \). The resulting success probability is \( \gamma(K + 1, H) \).

The end-to-end blocking probability is plotted against the converter position for \( K = 1 \) for two different link load patterns in Figure 4. The top curve is for uniform link loads and \( \rho = 0.1 \); and the bottom curve is for linearly increasing link loads from link 0 to link 9. We take \( \rho_0 = 0.05, \rho_0 = 0.1, \) and \( \rho_i = \rho_{i-1} + c, \ i = 1, \ldots, 8 \), where \( c = \frac{\rho_8 - \rho_0}{8} \). Intuitively the optimal converter location should shift to the right for the linear traffic pattern and this is indeed what is observed in the figure. However, it cannot be determined \textit{a priori} that the converter is optimally placed at node 6 for this traffic pattern. Note that, in the figure, converter placement in a non-optimal position would increase \( P_b \) by a factor of 2 at least and the performance degradation could be as high as two orders of magnitude.

![Figure 4: \( P_b \) vs. the converter position for \( K = 1 \) for uniform link loads (\( \rho = 0.1 \)) and for linearly increasing link loads (\( \rho_0 = 0.05, \rho_0 = 0.1 \)).](image)

The performances of random (R), uniform (U), and optimal (O) placement are compared for a linear link load traffic pattern in Figure 5. The uniform placement performs quite poorly for a large number of converters. This is due to the fact that converters are best placed towards the end of the path whereas uniform placement would place some towards the beginning of the path, where they are expected to be less useful. The optimal placement results in dramatically better performance than random placement.

2.3 Placement for Optimal Average Performance

Until this point, we have considered the placement problem from the point of view of a call going from node 0 to node \( H \). However, when a path is part of a large network, it may be of interest to minimize the average blocking probability over all traffic using the path, rather than just the end-to-end traffic. We now modify the solution above to take this into account. Random placement is considered first.
2.3.2 Optimal Placement

The optimal placement vector \( \mathbf{a}^{\text{opt}} \) that minimizes the average call blocking probability is given by \( \mathbf{a}^{\text{opt}} = \arg \max_{\mathbf{a} \in \Theta(K+1, H)} E_{(s,d)} \delta(H, \mathbf{a}, s, d) \). Note that the max and the \( E \) operators cannot be interchanged, and as a result, it is not possible to obtain a recurrence relation such as the one in (6). However, an approximate solution to this problem can be obtained as follows. Instead of maximizing \( E_{(s,d)} \delta(H, \mathbf{a}, s, d) \), we will obtain the placement configuration that maximizes \( E_{(s,d)} \ln \delta(H, \mathbf{a}, s, d) \). Note that \( 1 - \delta(H, \mathbf{a}, s, d) \) is the probability that the \( (s, d) \) call is blocked in the chain \([0, H]\) when the placement vector is \( \mathbf{a} \). For reasonably low blocking probabilities \(< 10^{-2}\), because of the fact that \( \ln x \approx x - 1 \) when \( x \approx 1 \), \( E_{(s,d)} \ln \delta(H, \mathbf{a}, s, d) \) is an excellent approximation for \( -E_{(s,d)} (1 - \delta(H, \mathbf{a}, s, d)) \) which is the original objective function to maximize.

Define \( \zeta(j, \mathbf{a}) \equiv E_{(s,d)} \ln \delta(j, \mathbf{a}, s, d) \). Now if \( \xi(m,j) \equiv \max_{\mathbf{a} \in \Theta(m,j)} \zeta(j, \mathbf{a}) \), then

\[
\xi(m,j) = \max_{1 \leq i \leq j - 1} \max_{\mathbf{a} \in \Theta(m,i)} \zeta(i, \mathbf{a})
\]

\[
= \max_{1 \leq i \leq j - 1} \{ \xi(m - 1, i) + E_{(s,d)} \ln h(i, j, s, d) \}
\]

(9)

for \( 2 \leq m \leq K + 1 \), \( m \leq j \leq H \), and \( \xi(1,j) = E_{(s,d)} \ln h(0, j, s, d) \) for \( 1 \leq j \leq H \).

The optimal placement \( \mathbf{a}^{\text{opt}} \) is now given by \( \arg \max_{\mathbf{a} \in \Theta(K+1, H)} \zeta(K + 1, \mathbf{a}) \) and (9) defines a recurrence relation that can be solved via dynamic programming as before. In Figure 6, we plot the average blocking probability for optimal and random placements and the following traffic pattern. The load per wavelength between each \((s, d)\) pair, \( \lambda_{sd} = 0.01 \), \( d > s \), so that the load per wavelength on link \( i \), \( \rho_i = 0.01 (i + 1) (H - i) \). In this traffic pattern, the link loads increase from link 0 linearly until link 4 and then decrease linearly from link 5 to link 9. We also plot the performance for an end-to-end call in the figure for comparison. We observe that optimal placement provides considerable improvement in average performance as well, relative to random placement. For instance, to achieve an average \( P_b \approx 10^{-3} \), 3 converters suffice if placed optimally while 7 converters would be required if placed randomly.

3 Converter Placement in a Bus

In the previous section, we considered the problem of optimally placing a given number of converters on a path assuming that link loads are independent. In this section, we show how that framework can be applied to a bus network where there is no interfering traffic [2] and the traffic in the network is solely due to the traffic originating at the nodes of the bus. In
such a scenario, the link load independence assumption of the previous section is not appropriate. We show in this section how our dynamic programming solutions are applicable even in the presence of link load correlation.

A key step in the derivation of the recurrence relation in (9) is the second equality where we have assumed that the success probability of an \((s, d)\) call in chain \([0, i]\) is independent of the success probability of the call in segment \([i, j]\) when node \(i\) is a converter. This assumption is still needed; that is, we continue to assume that the success probabilities of a call in disjoint segments are statistically independent. However, note that we do not need the independence assumption between the links of the same segment. Since the fraction of the nodes with converters is typically small, the most important effects of link load correlation will be taken into account in our formulation.

The performance model we use here is the one proposed by Barry in [2, 3]. For completeness, we present the relevant details of Barry’s model here. The only quantity of interest to us here is the probability of success \(f(i, j)\) in a subsegment \([i, j]\) since all other quantities are defined in terms of it.

Consider the bus in Figure 7 and let \(\rho_n(i)\) be the load per wavelength that enters the network at node \(i\), \(\rho_t(i)\) the load per wavelength that leaves the network at node \(i + 1\), and \(\rho_c(i)\) the load per wavelength that continues through node \(i + 1\) and uses links \(i\) and \(i + 1\). Also, let \(u\) denote the load per wavelength on link \(i\) by \(\rho(i)\).

Since wavelengths are considered to be independent of each other in this model, it suffices to look at a single wavelength, say \(\lambda_1\). Then, \(P_t(i) \equiv \rho_t(i)/\rho(i)\) is defined as the probability that a call on wavelength \(\lambda_1\) uses link \(i\) and leaves at node \(i + 1\). Also, let \(P_n(i)\) be the probability that a new call enters the network at node \(i\) and uses link \(i\) on wavelength \(\lambda_1\) given that \(\lambda_1\) is not used by another call on link \(i\). \(\rho(i)\) is then given by [2]

\[
\rho(i) = \rho(i - 1)[1 - P_t(i - 1) + P_t(i - 1)P_n(i)] + (1 - \rho(i - 1))P_n(i)
\]

and therefore \(P_n(i) = \frac{\rho(i) - \rho(i - 1)[1 - P_t(i - 1)]}{1 - \rho(i - 1)[1 - P_t(i - 1)]}\).

The probability that at least one wavelength is available on all links of a subsegment \([i, j]\) is now given by

\[
f(i, j) = 1 - \left[1 - \prod_{k=i+1}^{j-1} (1 - P_n(k))\right]^F. \tag{10}\]

Given the traffic matrix \(A = [\lambda_{sd}]\) where \(\lambda_{sd}\) is the load per wavelength between nodes \(s\) and \(d\), \(\rho_t(i)\), \(\rho_n(i)\), \(\rho_c(i)\), and \(\rho(i)\) are calculated in the following obvious way.

\[
\rho(i) = \sum_{s=0}^{i} \sum_{d=i+1}^{H} \lambda_{sd}, \quad \rho_t(i) = \sum_{s=0}^{i} \lambda_{s,i+1},\]

\[
\rho_n(i) = \sum_{d=i+1}^{H} \lambda_{id}, \quad \rho_c(i) = \rho(i) - \rho_t(i).
\]

As before, (7) and (9) are used to evaluate the performance under random and optimal placement, respectively. However, (10) is used instead of (5) for computing the success probability in a subsegment, thus taking the link load correlation in the bus topology into account.

We show \(P_L\) as a function of \(K\) for a 10-node bus with \(F = 10\) in Figure 8. The link loads are kept constant at \(\rho = 0.4\). This is done by setting \(\rho_n(0) = \rho\), and \(\rho_t(i) = \rho - \sum_{s=0}^{i-1} H(s)\rho_n(i - 1), i = 1, 2, \ldots, H - 1\), and \(\lambda_i = \frac{\rho_n(i)}{H(i)}\) for \(j > i\), and 0 otherwise. Here we observe that the performance improvement over random placement due to optimal converter placement is
not as high as in a path with independent link loads. However, the improvement is significant considering the fact that the performance does not improve substantially as the number of converters increases.

![Graph](image)

**Figure 8:** $P_b$ vs. $K$ for uniform link loads, $\rho = 0.4$.

Notice further that uniform converter placement is no longer optimal even though link loads are uniform, because of the link load correlation as well as the fact that we have considered the optimal placement with respect to all traffic and not just end-to-end traffic. $P_b$ vs. the converter position is plotted in Figure 9 for $K = 1$, and a uniform link load of $\rho = 0.3$. It is certainly not clear beforehand that the optimal solution would place the converter at node 7. This is in fact surprising considering the fact that node 7 is neither the node with the highest transit (continuing) traffic nor the node mixing the highest amount of traffic (high $\rho_n$ and $\rho_i$). For $K = 2$, the optimal placements turn out to be at nodes 5 and 8, again seemingly unintuitive.

We remark here that this formulation can be applied to a double-bus network as well. The only necessary change is that the objective function must now maximize the average success probability over both buses together. The traffic model will have to be applied to the two buses separately.

4 Converter Placement in a Ring

The ring is a popular optical network topology because of its simplicity and fault tolerance [4, 5, 6, 8]. In this section, we present a dynamic programming solution for optimal converter placement in a ring topology. As in the bus, the optimality criterion is the average blocking probability. We continue to assume that all wavelengths are used on a link with the same probability and wavelength events are independent of each other. However, the correlation between wavelength usage events on consecutive links will be taken into account as in the previous section.

Consider a unidirectional ring network with $H$ nodes, numbered from 0 to $H - 1$, where the link from node $i$ to node $(i + 1) \mod H$ is labeled as link $i$. The optimal placement for the ring is obtained by using the fact that if a converter is placed at a node, say $i$, then because of the assumed independence of the success probability of a call in disjoint segments, the ring can be logically broken at node $i$ to create two nodes $i'$ and $i''$ and creating a bus of length $H$. Let the left end-node of the resulting bus be $i'$ and the right end-node $i''$. The traffic matrix $\Lambda$ for the ring is modified to form a traffic matrix $\Lambda'$ for the bus as follows. $\lambda'_{i', j} = \lambda_{i, j}$ if chain $[s, d]$ contains node $i$ as an intermediate node in the ring, and $\lambda'_{i', j} = \lambda_{i, j}$ otherwise.

Given an $H$-node ring and $K$ converters, the average success probabilities can be obtained by conditioning on the lowest indexed node $i$ having a converter, $i = 0, 1, \ldots, H - K$. The ring is then broken at node $i$ as described above and the optimal placement of $K - 1$ converters on the logical bus given $i$ is the lowest indexed node with a converter is obtained as in the previous section. The optimal placement over the ring is the placement that maximizes the success probability over all $i$, and the performance with random placement is the performance averaged over all $i$.

We omit the details and present only the results.

4.1 Results

We consider a 10-node ring network with $F = 10$. Figure 10 shows $P_b$ vs. $K$ for optimal/uniform and random placement of converters when the link loads

![Graph](image)

**Figure 9:** $P_b$ vs. converter position for $K = 1$ and uniform link loads, $\rho = 0.3$. 

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are uniform (as are loads between any \((s, d)\) pair) with \(p = 0.4\). Again, due to the considerable load correlation, the performance improvement of optimal placement over random placement is only marginal compared to the gains achievable in a path where there is insignificant load correlation.

![Figure 10: \(P_b\) vs. \(K\) for uniform link loads, \(p = 0.4\).](image)

However, optimal placement could assume importance when the traffic is not uniform. For instance, if all traffic were local traffic (from a node to its neighbor) but for some traffic that goes from node 0 to node \(H - 1\), then the situation is similar to the one in the path with no load correlation. In that case, we observed that optimal placement can provide a significant improvement in end-to-end blocking performance (see Figure 2). Note that converters do not affect the blocking performance of local calls.

5 Conclusions

In this paper, we considered the problem of optimally placing a given number of converters in a path that is part of a dense network, in bus topologies, and in ring topologies. We first showed that uniformly spaced converters produce optimal performance for an end-to-end call on a path when the link loads are uncorrelated and uniform. When the link loads are non-uniform or when other calls are considered, we provided solutions based on dynamic programming for the optimal placement of converters. Expressions for blocking probability when the converters are randomly placed were also derived.

The results indicate that optimal converter placement is an important issue, especially when there is negligible correlation between link loads. Our solutions are applicable to a variety of traffic models, even though the results presented here were for the binomial model.

We have observed (results not shown in the paper) that the optimal placement is largely insensitive to the traffic model, but does depend on it at high blocking probabilities. We remark here that the optimal placement is not only somewhat dependent on the traffic model but also on the actual load when a given traffic pattern is scaled. That is, the optimal placements may change as the load is scaled up with the same traffic pattern. However, the changes do not appear to be arbitrary and there seem to be well-defined regions where the optimal placement configurations remain the same. This phenomenon needs further investigation.

Finally, the problem of optimally placing converters in an arbitrary topology is still an open problem and effective heuristics, possibly based on our solutions presented here, will be required.

References


